# Sensex Realized Volatility Index (REALVOL)

## Introduction

Volatility modelling has traditionally relied on complex econometric procedures in order to accommodate the inherent latent character of volatility. Realized volatility is an important metric that provides market participants an accurate measure of the historical volatility of the underlying over the life cycle of the derivative contract. Over the last decade, investors have extensively used volatility as a trading asset. The negative correlation between equity market returns and volatility has been well documented and thus volatility provides a significant diversification benefit to an investment portfolio.

The mechanics of the realized vol index are simple – we compute daily realized variance simply by summing squared returns. The theory of quadratic variation reveals that, under suitable conditions, realized volatility is not only an unbiased ex-post estimator of daily return volatility, but also asymptotically free of measurement error.

## Applications of Realized Volatility Index

- RVI Is considered a useful complement to the VIX because RVI captures realized volatility while the VIX measures implied volatility.
- Derivative contracts on RVI can be used for hedging gamma exposures and for directional bets on volatility.
- The skew needed to price out-of-the-money options can now be computed on a realized vs. implied basis
- With the advent of volatility and covariance swaps in the OTC market, realized volatility itself is now the underlying. Such swaps are useful for, among others, holders of options who wish to hedge their holdings, i.e., offset the impact of changes in volatility on the value of their positions.
- Improved volatility and correlation forecasts will also be useful for portfolio allocation and risk management.
- Swap contracts on realized variance have now been trading over the counter for some years with a fair degree of liquidity. More recently, derivatives whose payoffs are nonlinear functions of realized variance have also begun to trade over the counter. In particular, a natural outgrowth of the variance swap market is an interest in volatility swaps, which are essentially forward contracts written on the square root of realized variance.

## Section 1: Definition of Realized Variance Index & Realized Volatility Index

The formula for realized Variance uses continuously compounded daily returns assuming a mean daily price return of zero. The estimated variance is then annualized assuming 252 business days per year. **The realized volatility is the square root of the realized variance estimate**.

The following is the formula used to calculate the value of the SENSEX REALVOL index on the $n^{th}$ day of the index's underlying option expiry cycle:

$$\mathbf{REALVOL_n} = \sqrt{252 \times \left(\frac{\sum_{t=1}^{n} R_t^2}{n}\right)}$$

Where,

$n$ = $n^{th}$ day of the underlying option expiry cycle; resets to 1 at the start of a new cycle
$R_t = \ln (P_t/P_{t-1})$ = One-day log return of the SENSEX
$P_t$ = Closing value of the BSE SENSEX on the $t^{th}$ day of the option expiry cycle.

The realized volatility is the standard deviation of the daily log returns on the Sensex Index. However since the mean daily price return is zero, we use **"n" instead of "n-1"** in the denominator since the mean is not estimated.

**Rationale for assuming the mean daily return as Zero**

Here are the descriptive statistics for the daily returns on the Sensex from Jan 1, 2005 to Oct 31, 2010:

| ***Descriptive statistics of daily returns on Sensex - Jan 1, 2005 to Oct 31,2010*** | |
|---|---|
| Mean | 0.06% |
| Median | 0.14% |
| Standard Deviation | 1.97% |
| Kurtosis | 5.8574 |
| Skewness | 0.1011 |
| Sample Size | 1196 |
| Standard Error | 0.06% |
| **Test for the sample mean** | |
| T- Stat | 1.11 |
| **Jarque Bera test for Normality** | |
| JB stat | 1,686.03 |
| JB critical value(1% significance level) | 5.99 |
| JB P value | 0.0000 |

The empirical data reveals that the expected daily return is statistically not different from zero. The T-stat for the mean is 1.11 well below the critical value of 2.00 at the 95% confidence level. In other words the daily return observed is simply a manifestation of the volatility of the index. The return distribution exhibits leptokurtosis i.e. fat tails.  The Jarque Bera test for normality indicates that the underlying daily returns are non-normal.

**Refer: Appendix-1, 2 & 3 for a numerical example on the computation of Realized Variance and Realized Volatility Indices**

## Computation of Realized Variance and Realized Volatility Indices

**Expiry date for F&O contracts at BSE: Two Thursdays prior to the last Thursday of the month.**

**Different types of Realized Variance and Volatility Indices**

1) One month Realized Variance and Vol Indices
2) Two month Realized Variance and Vol Indices
3) Three month Realized Variance and Vol Indices

**Futures and Options contracts will be launched on the realized variance and realized volatility Indices after approval from the regulator.**

The one-month realized variance is calculated from a series of values of the SENSEX beginning with the closing price of the SENSEX on the first day of the one-month period, and ending with the closing price of the SENSEX on the last day of the one-month period.  The index will be reset for the next expiry cycle.

Consider the Nov expiry cycle for derivatives contracts. Derivative contracts at BSE expire on Thursday Nov 11, 2010.  The one month realized vol index will run from Nov 12, 2010 to Dec 16, 2010(i.e. expiry day for December 2010 derivatives contracts at BSE). The index will be reset and the next series will run from Dec 17, 2010 to Jan 13, 2011 and the process will be repeated on every expiry day.

The two month realized vol index will run from Nov 12, 2010 to Jan 13, 2011(i.e. expiry day for Jan 2011 derivatives contracts at BSE). The index will be reset and the next series will run from Jan 14, 2011 to March 17, 2011(i.e. expiry day for derivative contracts at BSE in March 2011).

The three month realized vol index will run from Nov 12, 2010 to Feb 11, 2011(i.e. expiry day for Feb 2011 derivatives contracts at BSE). The index will be reset and the next series will run from Feb 12, 2011 2010 to April 14, 2011(i.e. expiry day for derivative contracts at BSE in April 2011).

## Section 2: Hedging using derivatives contracts on realized volatility index

Let us look at the components of the profit and loss account of an option writer who is delta neutral. Delta refers to the first derivative sensitivity of the value of the option to the changes in the value of the underlying.

**For a Delta Neutral Option writer**

**Daily P&L on the short option delta neutral position =**

**Theta P&L + Gamma P&L + Vega P&L + residual P&L (i.e. influence of changes in interest rates and dividend expectations) – equation 1**

Gamma P&L refers to the manifestation of realized volatility, typically Gamma refers to the big unexpected jumps in underlying asset prices while Vega P&L refers to the impact of changes in implied volatilities.

**Eqn 1: Daily P&L =** $\frac{1}{2}\Gamma * (\Delta S)^2 + \Theta * (\Delta t) + \nu * (\Delta\sigma) + \ \ldots$

Where ΔS is the change in the price of the underlying, Δt reflects the fraction of time elapsed (Usually 1/365) and Δσ reflects the change in implied volatility.

For further analysis, we make the following assumptions

1) The residual P&L is negligible
2) Implied volatility term structure is flat

The assumptions reflect a Black Scholes world and the P&L equation simplifies to :

**Eqn 2: Daily P&L =** $\frac{1}{2}\Gamma * (\Delta S)^2 + \Theta * (\Delta t)$

Thus the daily P&L of a delta neutral option position is driven by theta and gamma.

Further there is a well established relationship between theta and gamma given below

**Eqn 3:** $\Theta \approx -\frac{1}{2}\Gamma S^2 \sigma^2$

Where S is the current spot price of the underlying and σ the current implied volatility of the option. Incorporating equation 3 in equation 2 and simplifying, we get

**Eqn 4: Daily P&L =** $\frac{1}{2}\Gamma S^2 * [\left(\frac{\Delta S}{S}\right)^2 - \sigma^2 \Delta t]$

The first term in the bracket reflects squared return of the underlying or the 1-day realized variance and the second term inside the bracket reflects the squared daily implied volatility. Thus the P&L of the delta hedged position is driven by the difference between realized and implied variance. Since variance is the square of volatility, **it is obvious that daily P&L is driven by the difference between realized volatility and implied volatility.**

An option writer makes gains when realized volatility is less than Implied Volatility i.e. when absolute value of Gamma P&L is less than absolute value of Vega P&L. An option writer incurs losses when realized volatility is greater than Implied Volatility i.e. when absolute value of Gamma P&L is greater than absolute value of Vega P&L.

The following example will illustrate that it is not good enough to be delta neutral, or in other words the option writer can suffer big losses when realized Volatility exceeds implied volatility.

**<u>Case1: Option finishes in the money</u>**

| | |
|---|---|
| Stock Price | 49 |
| Strike Price | 50 |
| Interest Rate | 5% |
| Time(Weeks) | 20 |
| Time(years) | 0.3846 |
| Volatility(annualized) | 20.00% |
| Dividend Yield | 0% |
| D1 | 0.0542 |
| Delta of the Call Option | 0.522 |
| # of Call Option Contracts sold | 1000 |
| Market Lot | 100 |
| # of shares corresponding to Option Position | 100000 |
| Black Scholes Value of the European Call | 2.40 |
| Value of the Option Position | 240000 |

A table illustrating the computation of the P&L on the position is given below:

| Week | Stock Price | Delta | Shares Purchased | Cost of Shares Purchased(000) | Interest Cost(000) | Cumulative Cost Including Interest(000) | P & L(000) |
|---|---|---|---|---|---|---|---|
| 0 | 49.00 | 0.522 | 52200 | 2557.8 | 2.500 | 2557.8 | 240.05 |
| 1 | 48.12 | 0.458 | -6400 | -308 | 2.200 | 2,252.3 | 191.65 |
| 2 | 47.37 | 0.400 | -5800 | -274.7 | 1.900 | 1,979.8 | 155.05 |
| 3 | 50.25 | 0.596 | 19600 | 984.9 | 2.800 | 2,966.6 | 253.45 |
| 4 | 51.75 | 0.693 | 9700 | 502 | 3.300 | 3,471.4 | 233.65 |
| 5 | 53.12 | 0.774 | 8100 | 430.3 | 3.700 | 3,905.0 | 205.05 |
| 6 | 53.00 | 0.771 | -300 | -15.9 | 3.700 | 3,892.8 | 202.25 |
| 7 | 51.87 | 0.706 | -6500 | -337.2 | 3.400 | 3,559.3 | 210.75 |
| 8 | 51.38 | 0.674 | -3200 | -164.4 | 3.200 | 3,398.3 | 211.75 |
| 9 | 53.00 | 0.787 | 11300 | 598.9 | 3.800 | 4,000.4 | 174.65 |
| 10 | 49.88 | 0.550 | -23700 | -1182.2 | 2.700 | 2,822.0 | 161.45 |

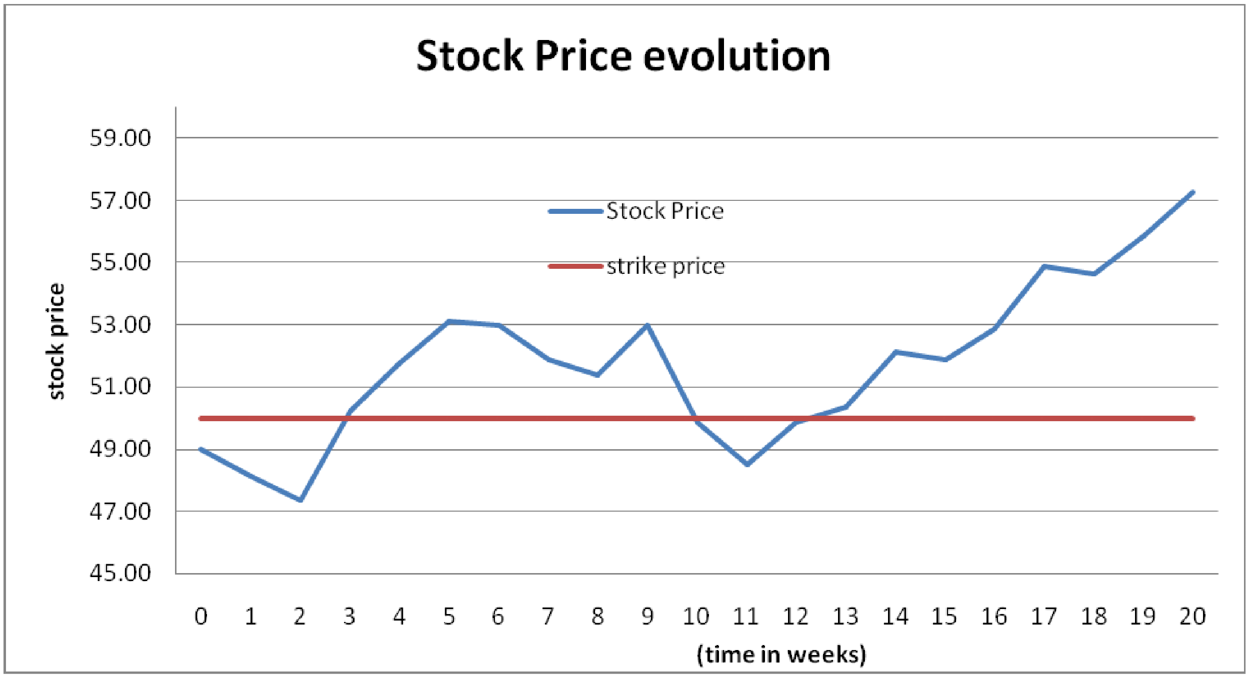
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 11 | 48.50 | 0.413 | -13700 | -664.5 | 2.000 | 2,160.2 | 82.90 |
| 12 | 49.88 | 0.542 | 12900 | 643.5 | 2.700 | 2,805.7 | 137.85 |
| 13 | 50.37 | 0.591 | 4900 | 246.8 | 2.900 | 3,055.2 | 139.85 |
| 14 | 52.13 | 0.768 | 17700 | 922.7 | 3.800 | 3,980.8 | 99.25 |
| 15 | 51.88 | 0.759 | -900 | -46.7 | 3.700 | 3,937.9 | 97.15 |
| 16 | 52.87 | 0.865 | 10600 | 560.4 | 4.300 | 4,502.0 | 63.05 |
| 17 | 54.87 | 0.978 | 11300 | 620 | 4.900 | 5,126.3 | 3.75 |
| 18 | 54.62 | 0.990 | 1200 | 65.5 | 4.900 | 5,196.7 | -6.65 |
| 19 | 55.87 | 1.000 | 1000 | 55.9 | 5.000 | 5,257.5 | -17.45 |
| 20 | 57.25 | 1.000 | 0 | 0.0 | 5.000 | 5,262.5 | -22.45 |

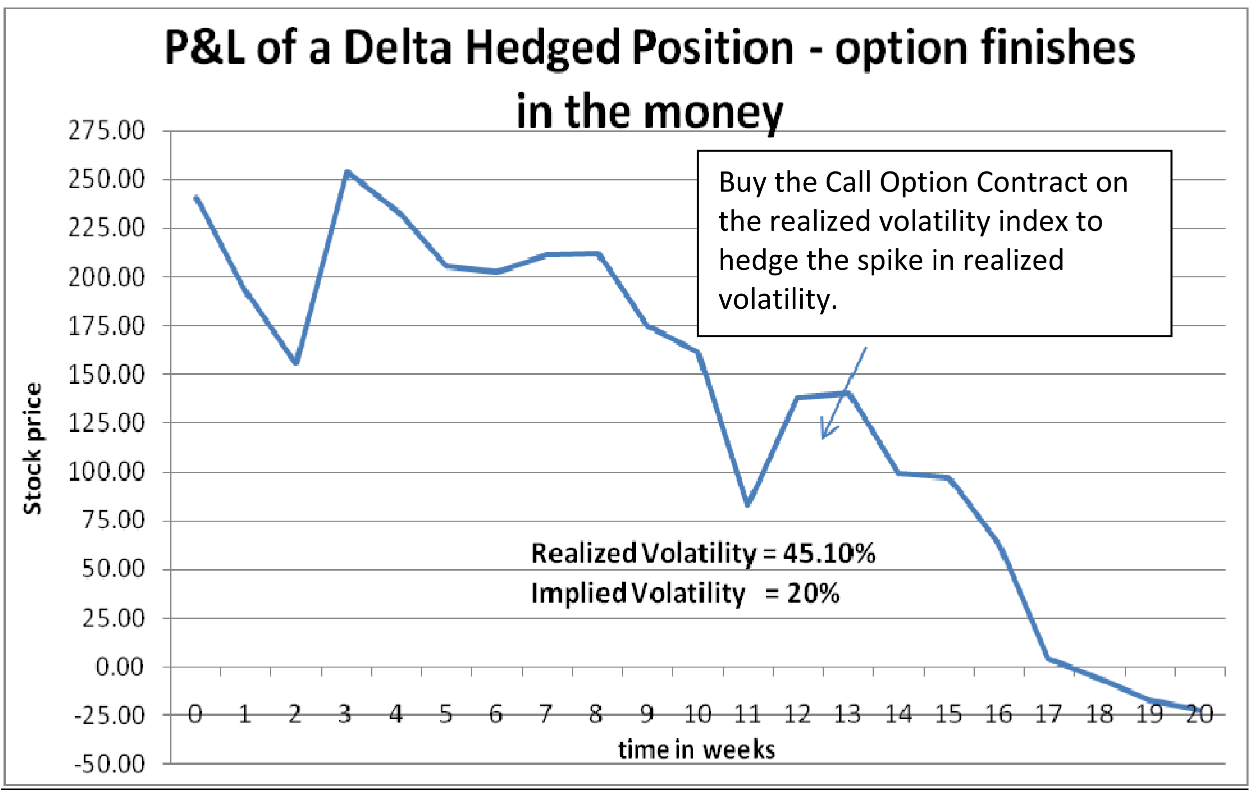
In this case the option finishes in the money at expiry. Delta hedging ensures that the option writer is fully covered i.e. owns 100% of the deliverable quantity (10000 shares in the example) on expiry. Delta hedging was not effective because:

a) When the underlying increases in value, the moneyness of the option increases and consequently the delta of the option increases. An increase in delta forces the option writer to buy more units of the underlying. Similarly when the underlying decreases in value, the delta of the option decreases or in other words the option writer will have to sell some units of the underlying in his inventory to rebalance the hedge. Thus delta hedging means "**Buy High Sell Low"** or in other words leads to a capital loss in the hedging activity.

b) The other costs include transaction costs and financing costs (assumed in this example at 5%) for investment in the underlying.

c) The option was priced at a volatility of **20%** while the realized volatility estimated on expiry was **45.10%. Thus the option writer was exposed to gamma risk, with the introduction of derivative contracts on realized volatility indices; the option writer can hedge his gamma by buying a call option on the realized volatility index.**


Stock Price evolution
59.00
57.00
55.00
53.00
51.00
49.00
47.00
45.00
stock price
Stock Price
strike price
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
(time in weeks)

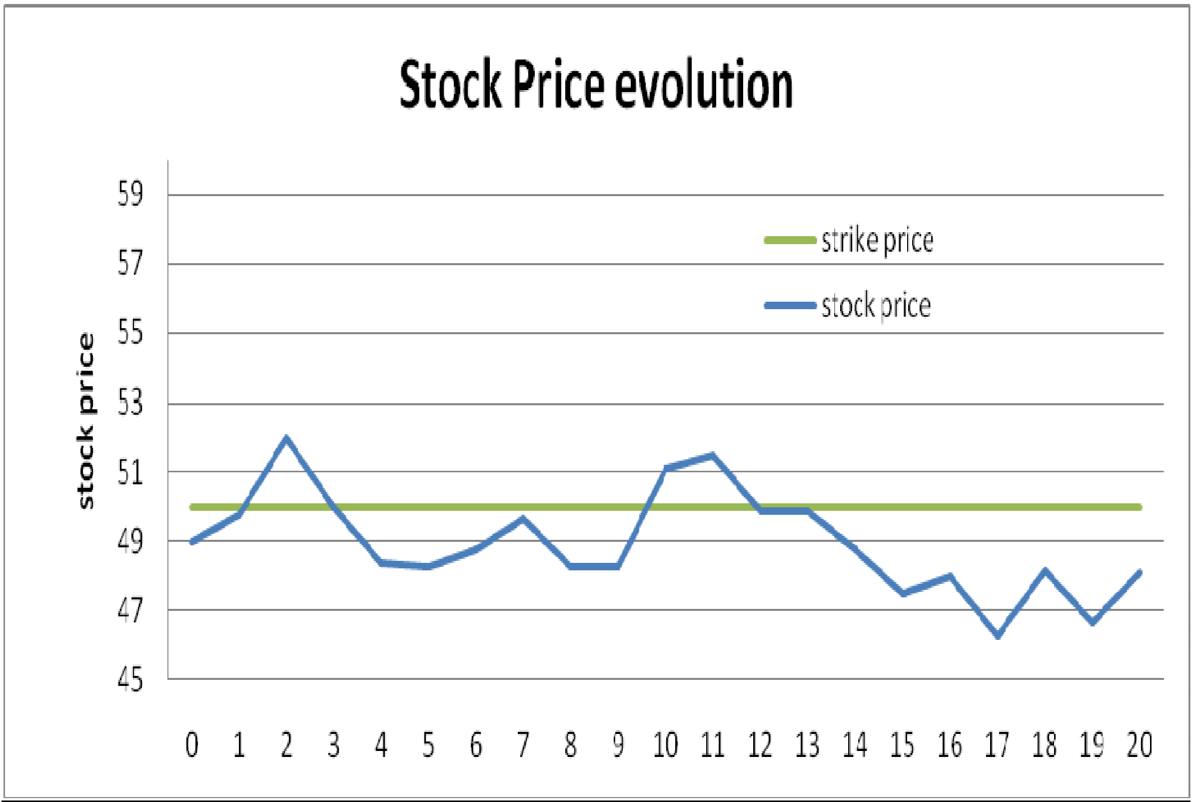

P&L of a Delta Hedged Position - option finishes in the money
275.00
250.00
225.00
200.00
175.00
150.00
125.00
100.00
75.00
50.00
25.00
0.00
-25.00
-50.00
Stock price
Buy the Call Option Contract on the realized volatility index to hedge the spike in realized volatility.
Realized Volatility = 45.10%
Implied Volatility = 20%
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
time in weeks

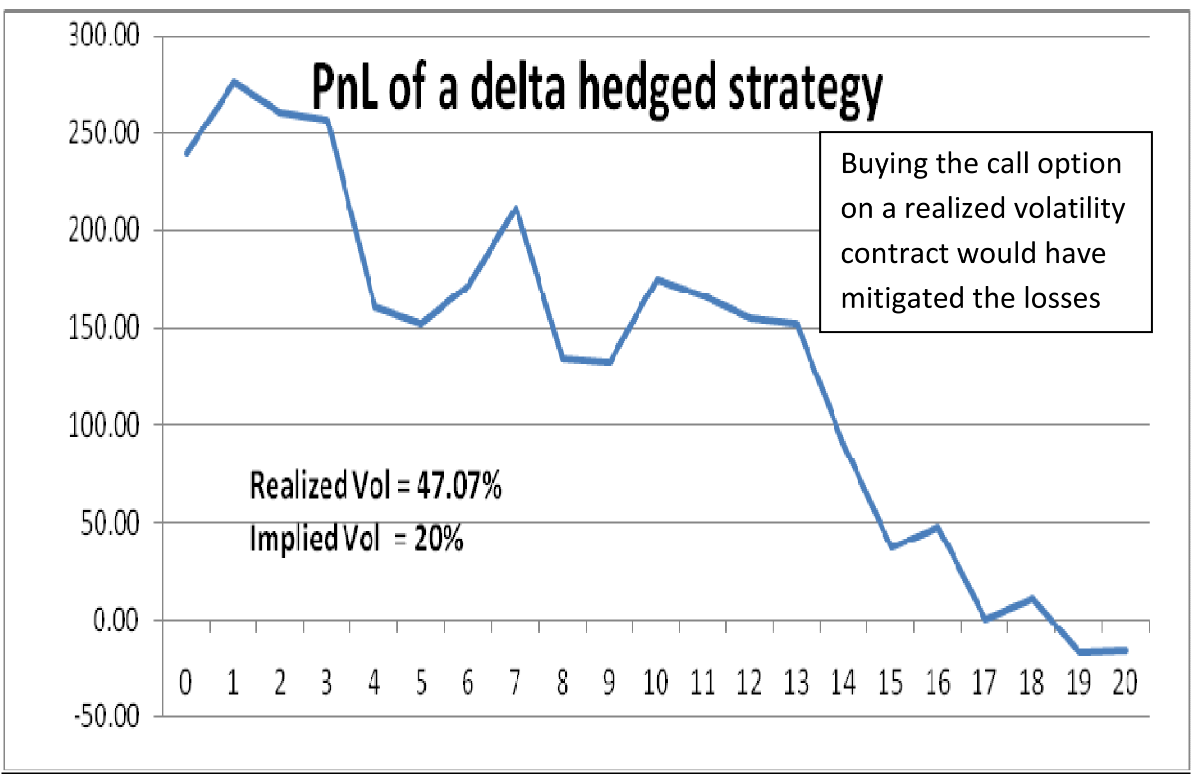
**Case 2: Option finishes out of the money**

The simulated stock price for the same example is given below:

| Week | Stock Price | Delta | Shares Purchased | Cost of Shares Purchased(000) | Interest Cost(000) | Cumulative Cost Including Interest(000) | P&L(000) |
|---|---|---|---|---|---|---|---|
| 0 | 49.00 | 0.522 | 52200 | 2557.8 | 2.500 | 2557.8 | 240.05 |
| 1 | 49.75 | 0.568 | 4600 | 228.9 | 2.700 | 2,789.2 | 276.65 |
| 2 | 52.00 | 0.705 | 13700 | 712.4 | 3.400 | 3,504.3 | 260.75 |
| 3 | 50.00 | 0.579 | -12600 | -630 | 2.800 | 2,877.7 | 257.35 |
| 4 | 48.38 | 0.459 | -12000 | -580.6 | 2.200 | 2,299.9 | 160.79 |
| 5 | 48.25 | 0.443 | -1600 | -77.2 | 2.100 | 2,224.9 | 152.63 |
| 6 | 48.75 | 0.475 | 3200 | 156 | 2.300 | 2,383.0 | 172.68 |
| 7 | 49.63 | 0.540 | 6500 | 322.6 | 2.600 | 2,707.9 | 212.17 |
| 8 | 48.25 | 0.420 | -12000 | -579 | 2.000 | 2,131.5 | 135.05 |
| 9 | 48.25 | 0.410 | -1000 | -48.3 | 2.000 | 2,085.2 | 133.10 |
| 10 | 51.12 | 0.658 | 24800 | 1267.8 | 3.200 | 3,355.0 | 175.05 |
| 11 | 51.50 | 0.692 | 3400 | 175.1 | 3.400 | 3,533.3 | 166.75 |
| 12 | 49.88 | 0.542 | -15000 | -748.2 | 2.700 | 2,788.5 | 155.05 |
| 13 | 49.88 | 0.538 | -400 | -20 | 2.600 | 2,771.2 | 152.40 |
| 14 | 48.75 | 0.400 | -13800 | -672.8 | 2.000 | 2,101.0 | 89.05 |
| 15 | 47.50 | 0.236 | -16400 | -779 | 1.200 | 1,324.0 | 37.05 |
| 16 | 48.00 | 0.261 | 2500 | 120 | 1.400 | 1,445.2 | 47.65 |
| 17 | 46.25 | 0.062 | -19900 | -920.4 | 0.500 | 526.2 | 0.60 |
| 18 | 48.13 | 0.183 | 12100 | 582.4 | 1.000 | 1,109.1 | 11.73 |
| 19 | 46.63 | 0.007 | -17600 | -820.7 | 0.200 | 289.4 | -16.71 |
| 20 | 48.12 | 0.000 | -700 | -33.7 | 0.200 | 255.9 | -15.85 |

**In this case the option finishes out of the money yet the option writer incurs a loss due to increased cost of hedging driven by manifestation of realized volatility. The realized volatility estimated on expiry was 47.07 %( annualized) while the option was priced at 20% volatility.**


Stock Price evolution
stock price
59
57
55
53
51
49
47
45
strike price
stock price
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20


PnL of a delta hedged strategy
300.00
250.00
200.00
150.00
100.00
50.00
0.00
-50.00
Buying the call option on a realized volatility contract would have mitigated the losses
Realized Vol = 47.07%
Implied Vol = 20%
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

## Section 3: Using derivatives on realized volatility indices to speculate on volatility

Consider a trader who feels that implied volatilities currently quoted in the market are too low and he/she reckons that volatilities are expected to spike up. The trader has few choices, most notable among them are:

- Buy a straddle or a strangle on the underlying.
- Buy a call option on the realized volatility index

Conversely if the speculative trader reckons that current implied volatility levels prevailing in the market are too high and he/she expects volatility levels to decline, then he/she can bet on volatility in different ways:

- Sell a straddle or strangle on the underlying
- Sell a call option or buy a put option on the realized volatility index.

**Thus derivative contracts on realized volatility indices offer a viable alternative to the current bouquet of option strategies which involve directional bets on volatility.**

## Section 4: Comparison of Realized Volatility Index and Implied Volatility Index

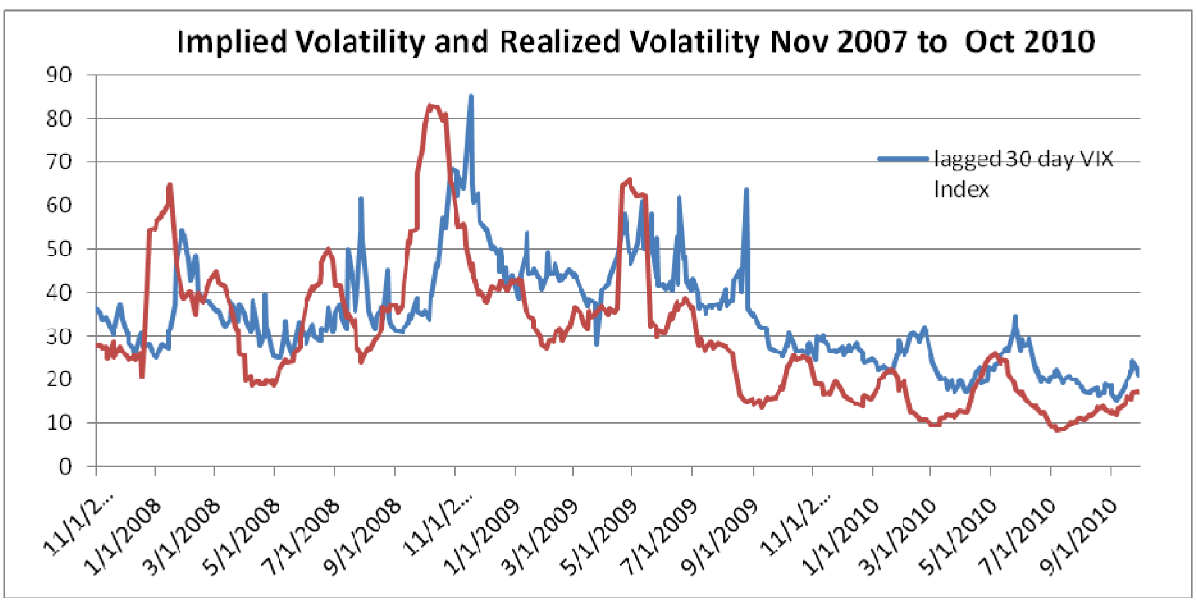
Implied volatility index such as the VIX published by NSE (hereafter referred to as the "NSE VIX Index") is a weighted average of implied volatilities of the options chain on the NIFTY index and is an estimate of the expected volatility for the next 30 calendar days.

Thus it is possible to explore the relationship between the VIX (ex-ante) and the realized volatility after 30 calendar days (ex-post). The difference between the two is an estimate of the forecast error.

Here is an example: The VIX Index on Nov 28, 2007 was 33.58 while the realized volatility one month later on Dec 28, 2007 was 26.72. Thus the prediction error was -6.86 i.e. the difference between the forecast volatility and the actually observed volatility.

Empirical research suggests that implied volatility tends to be invariably higher than subsequent realized volatility or in other words the option prices reflect a variance premium.

Let us look at the data from Nov 2007 when the VIX Index was introduced by NSE. Our sample data runs for close to three years from Nov 2007 to Oct 2010 and has a sample size of 418 observations.

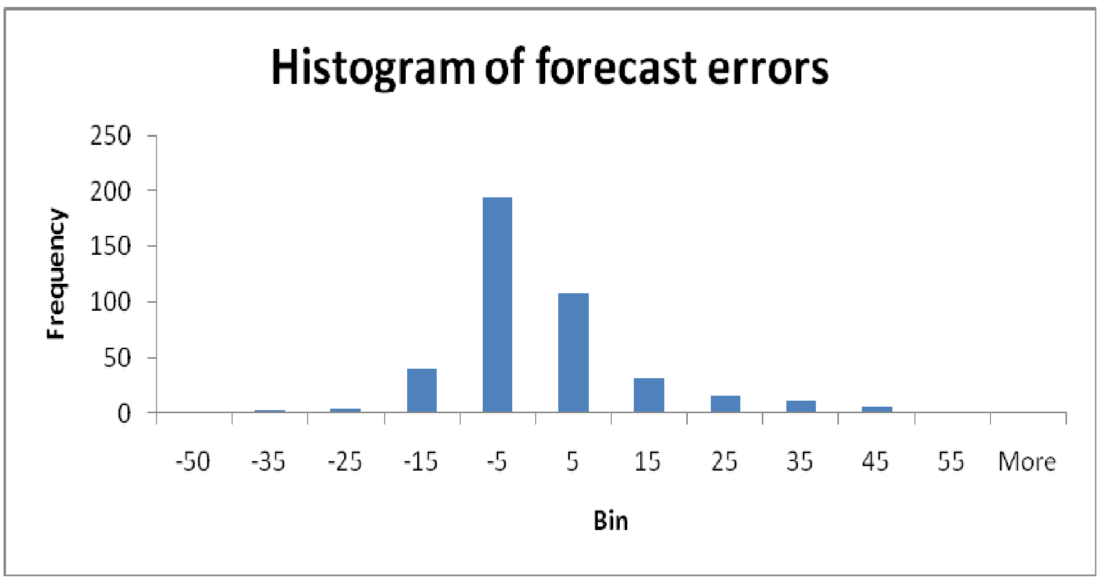



The graph confirms our intuition of variance premium in the market. Implied volatilities in Indian markets have been higher than subsequent realized volatilities for 80% of the observations in the sample.

**<u>The descriptive statistics for forecast errors (realized volatility- implied volatility) are given below:</u>**

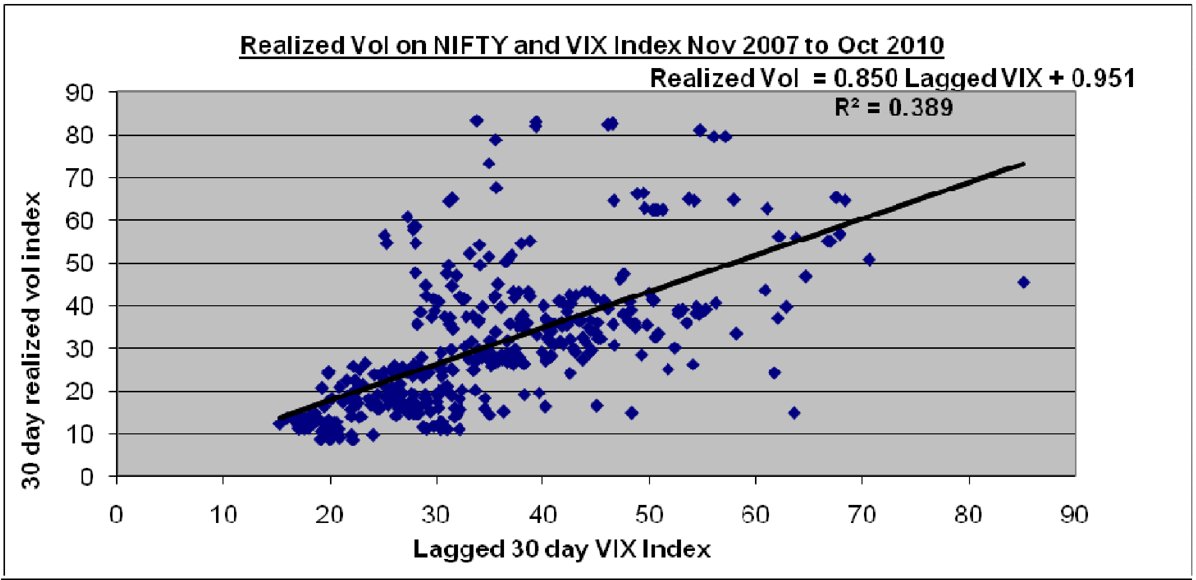
**Sample Period: Nov 12, 2007 to Oct 30, 2010**

| | |
|---|---|
| Mean | -4.1610 |
| Median | -6.4411 |
| Standard Deviation | 12.6520 |
| Kurtosis | 3.2584 |
| Skewness | 1.2450 |
| Range | 98.2388 |
| Minimum | -48.7443 |
| Maximum | 49.4945 |
| Sample Size | 418 |
| **% of negative observations** | **79.43%** |



**Is the Implied Volatility (VIX) an unbiased estimator of the realized volatility?**

Let us look at the scatter plot of the 30-day realized volatility and the lagged 30-day value of the VIX index which is the predictor variable.




We run the regression: $\mathbf{RV_t = \alpha_0 + \alpha_1\ VIX_{t\text{-}30} + \xi_t}$

Where $RV_t$ refers to the realized volatility of the NIFTY Index at time't' and the $VIX_{t\text{-}30}$ refers to the lagged 30-day value of the VIX Index

If the VIX is an unbiased estimator of the 30 day realized volatility, then $\alpha_0 = 0$ and $\alpha_1 = 1$

The regression output reveals that the intercept term $\alpha_0 = 0$ since its T-Stat is 0.1934 well below the critical value of 2.00 at the 95% confidence level.

**Regression Output:**

| | Coefficients | Standard Error | t Stat |
|---|---|---|---|
| Intercept | 0.9514 | 1.8852 | 0.5046 |
| Beta for the Lagged VIX | 0.8502 | 0.05223 | 16.2765 |

| Regression Statistics | |
|---|---|
| R Squared | 0.3890 |
| Standard Error | 12.5438 |
| Observations | 418 |
| F- stat | 264.9247 |
| F Critical value at 1% significance level | 4.6565 |

The intercept term ($\alpha_0$) is statistically not different from zero.

**Test for slope coefficient $\alpha_1$= 1**

| **T-test for slope coefficient = 1** | |
|---|---|
| T Stat = ( estimate -1) / Standard error | 0.9341 |
| T- Critical value at 95% confidence level | 1.9656 |
| Sample Size | 420 |

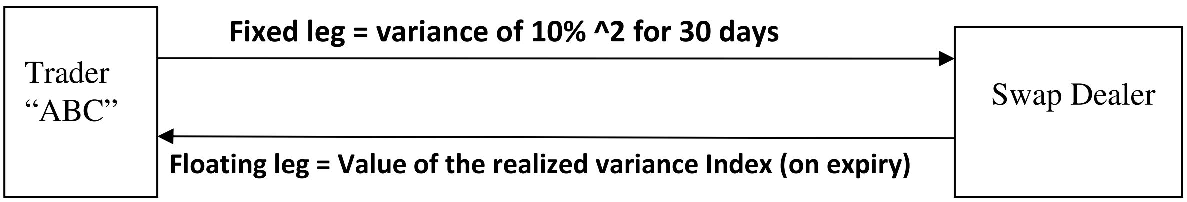
We reject the null hypothesis that the slope coefficient **(α1) = 1** and the intercept term is statistically equal to zero.

Since the slope term of the coefficient (α1) is different from "1" we conclude that "**Implied volatility is a biased estimator of the subsequent realized volatility"**

## Section 5: Swap contracts on realized variance i.e. variance swaps

Start Date of the Swap: Sept 16, 2010
End date of the Swap = Oct 14, 2010

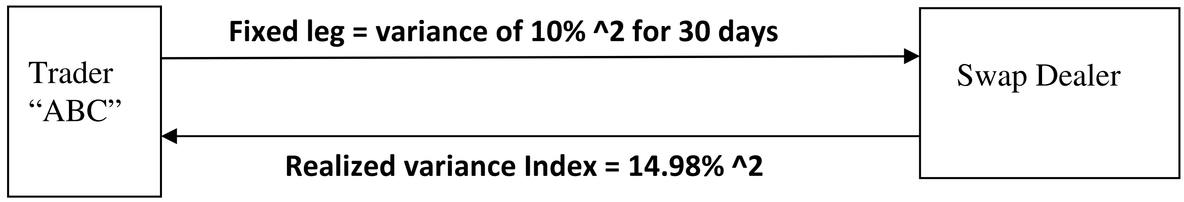
In the aforesaid example, the trader "ABC" will pay a fixed variance of 10% ^2 and receive the value of the realized variance index on Oct 14, 2010.




Let us look at how the variance swap is constructed. Trader "ABC" has a directional view on volatility. He/she reckons that current volatility levels (i.e. levels on Sept 16, 2010) are too low and the volatility is expected to spike up. Thus he/she enters into a variance swap to pay fixed and receive floating. The floating leg of the swap refers to the value of the realized variance index on expiry. Let us assume the realized volatility index on Oct 14, 2010 was 15% which means the realized variance index was 14.98 % ^2. See calculations below:

| Date | Close | R | R^2 | ∑ R^2 | count | Realized Variance | Realized Volatility |
|---|---|---|---|---|---|---|---|
| 16-Sep-10 | 19417.49 | | | | | | |
| 17-Sep-10 | 19594.75 | 0.0091 | 0.00008 | 0.00008 | 1 | 201.60 | 14.20 |
| 20-Sep-10 | 19906.1 | 0.0158 | 0.00025 | 0.00033 | 2 | 415.80 | 20.39 |
| 21-Sep-10 | 20001.55 | 0.0048 | 0.00002 | 0.00035 | 3 | 294.00 | 17.15 |
| 22-Sep-10 | 19941.72 | -0.003 | 0.00001 | 0.00036 | 4 | 226.80 | 15.06 |
| 23-Sep-10 | 19861.01 | -0.0041 | 0.00002 | 0.00038 | 5 | 191.52 | 13.84 |
| 24-Sep-10 | 20045.18 | 0.0092 | 0.00008 | 0.00046 | 6 | 193.20 | 13.90 |
| 27-Sep-10 | 20117.38 | 0.0036 | 0.00001 | 0.00047 | 7 | 169.20 | 13.01 |
| 28-Sep-10 | 20104.86 | -0.0006 | 0.00000 | 0.00047 | 8 | 148.05 | 12.17 |
| 29-Sep-10 | 19956.34 | -0.0074 | 0.00005 | 0.00052 | 9 | 145.60 | 12.07 |
| 30-Sep-10 | 20069.12 | 0.0056 | 0.00003 | 0.00055 | 10 | 138.60 | 11.77 |
| 1-Oct-10 | 20445.04 | 0.0186 | 0.00035 | 0.00090 | 11 | 206.18 | 14.36 |
| 4-Oct-10 | 20475.73 | 0.0015 | 0.00000 | 0.00090 | 12 | 189.00 | 13.75 |
| 5-Oct-10 | 20407.71 | -0.0033 | 0.00001 | 0.00091 | 13 | 176.40 | 13.28 |
| 6-Oct-10 | 20543.08 | 0.0066 | 0.00004 | 0.00095 | 14 | 171.00 | 13.08 |
| 7-Oct-10 | 20315.32 | -0.0111 | 0.00012 | 0.00107 | 15 | 179.76 | 13.41 |
| 8-Oct-10 | 20250.26 | -0.0032 | 0.00001 | 0.00108 | 16 | 170.10 | 13.04 |
| 11-Oct-10 | 20339.89 | 0.0044 | 0.00002 | 0.00110 | 17 | 163.06 | 12.77 |
| 12-Oct-10 | 20203.34 | -0.0067 | 0.00004 | 0.00114 | 18 | 159.60 | 12.63 |
| 13-Oct-10 | 20687.88 | 0.0237 | 0.00056 | 0.00170 | 19 | 225.47 | 15.02 |
| 14-Oct-10 | 20497.64 | -0.0092 | 0.00008 | 0.00178 | 20 | 224.28 | 14.98 |

**Cash Flows of the Swap on expiry**




Variance swaps and volatility swaps can also be used for hedging by option writers. Option writers who wish to hedge “Gamma exposures” will receive realized variance (floating leg) and pay fixed variance to the swap dealer.

Variance swaps and volatility swaps provide pure exposure to the volatility of the underlying asset. Traders can bet on volatility using option combinations like straddles and strangles, however an options combination position will require constant delta hedging so that the direction risk of the underlying asset is removed.  On the contrary,the profit and loss of a variance swap depends only on the difference between realized and implied volatility.

## Section 6: Conclusion

The realized volatility index(REALVOL) using the Sensex as an underlying provides traders with a valuable tool to hedge risk exposures and speculate on volatility using derivative contracts on the index or by using variance and volatility swaps in the OTC market. Empirical studies reveal that the implied volatility is a biased and inefficient estimator of future realized volatility. Our statistical tests validate this empirical finding for the

The Sensex and NIFTY indices are highly correlated and the correlation estimate of daily returns is close to 0.99. Products based on realized volatility index can be used by traders to hedge and speculate on volatility of the NIFTY index. The daily P&L of a delta neutral hedge is driven by the difference between the realized volatility on expiry and the implied volatility of the option position. Thus traders can hedge Vega exposures using derivatives based on the VIX index and can hedge their gamma exposures using derivatives on the realized volatility index.

## Appendix -1

One month realized volatility index: for Aug 2010 expiry cycle

Start date: July 15, 2010

End date: Aug 12, 2010

Variance = 10000 * $\sum R_i^{2}$ * 252 / Count. Rounded to two decimal places

Volatility = √variance. Rounded to two decimal places

| Date | Close | R | R^2 | ∑ R^2 | count | Realized Variance index | Realized Volatility index |
|---|---|---|---|---|---|---|---|
| 15-Jul-10 | 17909.46 | | | | | | |
| 16-Jul-10 | 17955.82 | 0.0026 | 0.00001 | 0.00001 | 1 | 25.20 | 5.02 |
| 19-Jul-10 | 17928.42 | -0.0015 | 0.00000 | 0.00001 | 2 | 12.60 | 3.55 |
| 20-Jul-10 | 17878.14 | -0.0028 | 0.00001 | 0.00002 | 3 | 16.80 | 4.10 |
| 21-Jul-10 | 17977.23 | 0.0055 | 0.00003 | 0.00005 | 4 | 31.50 | 5.61 |
| 22-Jul-10 | 18113.15 | 0.0075 | 0.00006 | 0.00011 | 5 | 55.44 | 7.45 |
| 23-Jul-10 | 18130.98 | 0.001 | 0.00000 | 0.00011 | 6 | 46.20 | 6.80 |
| 26-Jul-10 | 18020.05 | -0.0061 | 0.00004 | 0.00015 | 7 | 54.00 | 7.35 |
| 27-Jul-10 | 18077.61 | 0.0032 | 0.00001 | 0.00016 | 8 | 50.40 | 7.10 |
| 28-Jul-10 | 17957.37 | -0.0067 | 0.00004 | 0.00020 | 9 | 56.00 | 7.48 |
| 29-Jul-10 | 17992 | 0.0019 | 0.00000 | 0.00020 | 10 | 50.40 | 7.10 |
| 30-Jul-10 | 17868.29 | -0.0069 | 0.00005 | 0.00025 | 11 | 57.27 | 7.57 |
| 2-Aug-10 | 18081.21 | 0.0118 | 0.00014 | 0.00039 | 12 | 81.90 | 9.05 |
| 3-Aug-10 | 18114.83 | 0.0019 | 0.00000 | 0.00039 | 13 | 75.60 | 8.69 |
| 4-Aug-10 | 18217.44 | 0.0056 | 0.00003 | 0.00042 | 14 | 75.60 | 8.69 |
| 5-Aug-10 | 18172.83 | -0.0025 | 0.00001 | 0.00043 | 15 | 72.24 | 8.50 |
| 6-Aug-10 | 18143.99 | -0.0016 | 0.00000 | 0.00043 | 16 | 67.73 | 8.23 |
| 9-Aug-10 | 18287.5 | 0.0079 | 0.00006 | 0.00049 | 17 | 72.64 | 8.52 |
| 10-Aug-10 | 18219.99 | -0.0037 | 0.00001 | 0.00050 | 18 | 70.00 | 8.37 |
| 11-Aug-10 | 18070.19 | -0.0083 | 0.00007 | 0.00057 | 19 | 75.60 | 8.69 |
| 12-Aug-10 | 18073.9 | 0.0002 | 0.00000 | 0.00057 | 20 | 71.82 | 8.47 |

## **Appendix -2**

Two month realized volatility index: for Sept 2010 expiry cycle

Start date: July 15, 2010

End date: Sept 16, 2010

Variance = 10000 * $\sum R_i^{2}$ * 252 / Count. Rounded to two decimal places

Volatility = √variance. Rounded to two decimal places

| Date | Close | R | R^2 | ∑ R^2 | count | Realized Variance | Realized Vol |
|---|---|---|---|---|---|---|---|
| 15-Jul-10 | 17909.46 | | | | | | |
| 16-Jul-10 | 17955.82 | 0.0026 | 0.00001 | 0.000007 | 1 | 16.84 | 4.1 |
| 19-Jul-10 | 17928.42 | -0.0015 | 0.00000 | 0.000009 | 2 | 11.36 | 3.37 |
| 20-Jul-10 | 17878.14 | -0.0028 | 0.00001 | 0.000017 | 3 | 14.2 | 3.77 |
| 21-Jul-10 | 17977.23 | 0.0055 | 0.00003 | 0.000047 | 4 | 29.9 | 5.47 |
| 22-Jul-10 | 18113.15 | 0.0075 | 0.00006 | 0.000104 | 5 | 52.51 | 7.25 |
| 23-Jul-10 | 18130.98 | 0.0010 | 0.00000 | 0.000105 | 6 | 44.17 | 6.65 |
| 26-Jul-10 | 18020.05 | -0.0061 | 0.00004 | 0.000143 | 7 | 51.41 | 7.17 |
| 27-Jul-10 | 18077.61 | 0.0032 | 0.00001 | 0.000153 | 8 | 48.19 | 6.94 |
| 28-Jul-10 | 17957.37 | -0.0067 | 0.00004 | 0.000198 | 9 | 55.31 | 7.44 |
| 29-Jul-10 | 17992 | 0.0019 | 0.00000 | 0.000201 | 10 | 50.71 | 7.12 |
| 30-Jul-10 | 17868.29 | -0.0069 | 0.00005 | 0.000249 | 11 | 57.01 | 7.55 |
| 2-Aug-10 | 18081.21 | 0.0118 | 0.00014 | 0.000389 | 12 | 81.72 | 9.04 |
| 3-Aug-10 | 18114.83 | 0.0019 | 0.00000 | 0.000393 | 13 | 76.11 | 8.72 |
| 4-Aug-10 | 18217.44 | 0.0056 | 0.00003 | 0.000425 | 14 | 76.41 | 8.74 |
| 5-Aug-10 | 18172.83 | -0.0025 | 0.00001 | 0.000431 | 15 | 72.33 | 8.5 |
| 6-Aug-10 | 18143.99 | -0.0016 | 0.00000 | 0.000433 | 16 | 68.21 | 8.26 |
| 9-Aug-10 | 18287.5 | 0.0079 | 0.00006 | 0.000495 | 17 | 73.39 | 8.57 |
| 10-Aug-10 | 18219.99 | -0.0037 | 0.00001 | 0.000509 | 18 | 71.23 | 8.44 |
| 11-Aug-10 | 18070.19 | -0.0083 | 0.00007 | 0.000577 | 19 | 76.52 | 8.75 |
| 12-Aug-10 | 18073.9 | 0.0002 | 0.00000 | 0.000577 | 20 | 72.7 | 8.53 |
| 13-Aug-10 | 18167.03 | 0.0051 | 0.00003 | 0.000603 | 21 | 72.41 | 8.51 |
| 16-Aug-10 | 18050.78 | -0.0064 | 0.00004 | 0.000645 | 22 | 73.84 | 8.59 |
| 17-Aug-10 | 18048.85 | -0.0001 | 0.00000 | 0.000645 | 23 | 70.63 | 8.4 |
| 18-Aug-10 | 18257.12 | 0.0115 | 0.00013 | 0.000776 | 24 | 81.51 | 9.03 |
| 19-Aug-10 | 18454.94 | 0.0108 | 0.00012 | 0.000892 | 25 | 89.95 | 9.48 |
| 20-Aug-10 | 18401.82 | -0.0029 | 0.00001 | 0.000901 | 26 | 87.3 | 9.34 |
| 23-Aug-10 | 18409.35 | 0.0004 | 0.00000 | 0.000901 | 27 | 84.08 | 9.17 |
| 24-Aug-10 | 18311.59 | -0.0053 | 0.00003 | 0.000929 | 28 | 83.63 | 9.14 |
| 25-Aug-10 | 18179.64 | -0.0072 | 0.00005 | 0.000982 | 29 | 85.29 | 9.24 |
| 26-Aug-10 | 18226.35 | 0.0026 | 0.00001 | 0.000988 | 30 | 83 | 9.11 |
| 27-Aug-10 | 17998.41 | -0.0126 | 0.00016 | 0.001147 | 31 | 93.2 | 9.65 |
| 30-Aug-10 | 18032.11 | 0.0019 | 0.00000 | 0.001150 | 32 | 90.56 | 9.52 |
| 31-Aug-10 | 17971.12 | -0.0034 | 0.00001 | 0.001161 | 33 | 88.69 | 9.42 |
| 1-Sep-10 | 18205.87 | 0.0130 | 0.00017 | 0.001330 | 34 | 98.57 | 9.93 |
| 2-Sep-10 | 18238.31 | 0.0018 | 0.00000 | 0.001333 | 35 | 95.98 | 9.8 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3-Sep-10 | 18221.43 | -0.0009 | 0.00000 | 0.001334 | 36 | 93.38 | 9.66 |
| 6-Sep-10 | 18560.05 | 0.0184 | 0.00034 | 0.001673 | 37 | 113.94 | 10.67 |
| 7-Sep-10 | 18645.06 | 0.0046 | 0.00002 | 0.001694 | 38 | 112.33 | 10.6 |
| 8-Sep-10 | 18666.71 | 0.0012 | 0.00000 | 0.001695 | 39 | 109.54 | 10.47 |
| 9-Sep-10 | 18799.66 | 0.0071 | 0.00005 | 0.001746 | 40 | 109.97 | 10.49 |
| 13-Sep-10 | 19208.33 | 0.0215 | 0.00046 | 0.002208 | 41 | 135.71 | 11.65 |
| 14-Sep-10 | 19346.96 | 0.0072 | 0.00005 | 0.002260 | 42 | 135.59 | 11.64 |
| 15-Sep-10 | 19502.11 | 0.0080 | 0.00006 | 0.002324 | 43 | 136.17 | 11.67 |
| 16-Sep-10 | 19417.49 | -0.0043 | 0.00002 | 0.002342 | 44 | 134.16 | 11.58 |

## Appendix -3

Two month realized volatility index: for Sept 2010 expiry cycle

Start date: July 15, 2010

End date: Oct 14, 2010

Variance = 10000 * $\sum R_i^{\wedge 2}$ * 252 / Count. Rounded to two decimal places

Volatility = √variance. Rounded to two decimal places

| Date | Close | R | R^2 | ∑ R^2 | count | Realized Variance | Realized Volatility index |
|---|---|---|---|---|---|---|---|
| 15-Jul-10 | 17909.46 | | | | | | |
| 16-Jul-10 | 17955.82 | 0.0026 | 0.00001 | 0.00001 | 1 | 16.84 | 4.10 |
| 19-Jul-10 | 17928.42 | -0.0015 | 0.00000 | 0.00001 | 2 | 11.36 | 3.37 |
| 20-Jul-10 | 17878.14 | -0.0028 | 0.00001 | 0.00002 | 3 | 14.2 | 3.77 |
| 21-Jul-10 | 17977.23 | 0.0055 | 0.00003 | 0.00005 | 4 | 29.9 | 5.47 |
| 22-Jul-10 | 18113.15 | 0.0075 | 0.00006 | 0.00010 | 5 | 52.51 | 7.25 |
| 23-Jul-10 | 18130.98 | 0.0010 | 0.00000 | 0.00011 | 6 | 44.17 | 6.65 |
| 26-Jul-10 | 18020.05 | -0.0061 | 0.00004 | 0.00014 | 7 | 51.41 | 7.17 |
| 27-Jul-10 | 18077.61 | 0.0032 | 0.00001 | 0.00015 | 8 | 48.19 | 6.94 |
| 28-Jul-10 | 17957.37 | -0.0067 | 0.00004 | 0.00020 | 9 | 55.31 | 7.44 |
| 29-Jul-10 | 17992 | 0.0019 | 0.00000 | 0.00020 | 10 | 50.71 | 7.12 |
| 30-Jul-10 | 17868.29 | -0.0069 | 0.00005 | 0.00025 | 11 | 57.01 | 7.55 |
| 2-Aug-10 | 18081.21 | 0.0118 | 0.00014 | 0.00039 | 12 | 81.72 | 9.04 |
| 3-Aug-10 | 18114.83 | 0.0019 | 0.00000 | 0.00039 | 13 | 76.11 | 8.72 |
| 4-Aug-10 | 18217.44 | 0.0056 | 0.00003 | 0.00042 | 14 | 76.41 | 8.74 |
| 5-Aug-10 | 18172.83 | -0.0025 | 0.00001 | 0.00043 | 15 | 72.33 | 8.50 |
| 6-Aug-10 | 18143.99 | -0.0016 | 0.00000 | 0.00043 | 16 | 68.21 | 8.26 |
| 9-Aug-10 | 18287.5 | 0.0079 | 0.00006 | 0.00050 | 17 | 73.39 | 8.57 |
| 10-Aug-10 | 18219.99 | -0.0037 | 0.00001 | 0.00051 | 18 | 71.23 | 8.44 |
| 11-Aug-10 | 18070.19 | -0.0083 | 0.00007 | 0.00058 | 19 | 76.52 | 8.75 |
| 12-Aug-10 | 18073.9 | 0.0002 | 0.00000 | 0.00058 | 20 | 72.7 | 8.53 |
| 13-Aug-10 | 18167.03 | 0.0051 | 0.00003 | 0.00060 | 21 | 72.41 | 8.51 |
| 16-Aug-10 | 18050.78 | -0.0064 | 0.00004 | 0.00064 | 22 | 73.84 | 8.59 |
| 17-Aug-10 | 18048.85 | -0.0001 | 0.00000 | 0.00064 | 23 | 70.63 | 8.40 |
| 18-Aug-10 | 18257.12 | 0.0115 | 0.00013 | 0.00078 | 24 | 81.51 | 9.03 |
| 19-Aug-10 | 18454.94 | 0.0108 | 0.00012 | 0.00089 | 25 | 89.95 | 9.48 |
| 20-Aug-10 | 18401.82 | -0.0029 | 0.00001 | 0.00090 | 26 | 87.3 | 9.34 |
| 23-Aug-10 | 18409.35 | 0.0004 | 0.00000 | 0.00090 | 27 | 84.08 | 9.17 |
| 24-Aug-10 | 18311.59 | -0.0053 | 0.00003 | 0.00093 | 28 | 83.63 | 9.14 |
| 25-Aug-10 | 18179.64 | -0.0072 | 0.00005 | 0.00098 | 29 | 85.29 | 9.24 |
| 26-Aug-10 | 18226.35 | 0.0026 | 0.00001 | 0.00099 | 30 | 83 | 9.11 |
| 27-Aug-10 | 17998.41 | -0.0126 | 0.00016 | 0.00115 | 31 | 93.2 | 9.65 |
| 30-Aug-10 | 18032.11 | 0.0019 | 0.00000 | 0.00115 | 32 | 90.56 | 9.52 |
| 31-Aug-10 | 17971.12 | -0.0034 | 0.00001 | 0.00116 | 33 | 88.69 | 9.42 |
| 1-Sep-10 | 18205.87 | 0.0130 | 0.00017 | 0.00133 | 34 | 98.57 | 9.93 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2-Sep-10 | 18238.31 | 0.0018 | 0.00000 | 0.00133 | 35 | 95.98 | 9.80 |
| 3-Sep-10 | 18221.43 | -0.0009 | 0.00000 | 0.00133 | 36 | 93.38 | 9.66 |
| 6-Sep-10 | 18560.05 | 0.0184 | 0.00034 | 0.00167 | 37 | 113.94 | 10.67 |
| 7-Sep-10 | 18645.06 | 0.0046 | 0.00002 | 0.00169 | 38 | 112.33 | 10.60 |
| 8-Sep-10 | 18666.71 | 0.0012 | 0.00000 | 0.00170 | 39 | 109.54 | 10.47 |
| 9-Sep-10 | 18799.66 | 0.0071 | 0.00005 | 0.00175 | 40 | 109.97 | 10.49 |
| 13-Sep-10 | 19208.33 | 0.0215 | 0.00046 | 0.00221 | 41 | 135.71 | 11.65 |
| 14-Sep-10 | 19346.96 | 0.0072 | 0.00005 | 0.00226 | 42 | 135.59 | 11.64 |
| 15-Sep-10 | 19502.11 | 0.0080 | 0.00006 | 0.00232 | 43 | 136.17 | 11.67 |
| 16-Sep-10 | 19417.49 | -0.0043 | 0.00002 | 0.00234 | 44 | 134.16 | 11.58 |
| 17-Sep-10 | 19594.75 | 0.0091 | 0.00008 | 0.00243 | 45 | 135.8 | 11.65 |
| 20-Sep-10 | 19906.1 | 0.0158 | 0.00025 | 0.00267 | 46 | 146.47 | 12.10 |
| 21-Sep-10 | 20001.55 | 0.0048 | 0.00002 | 0.00270 | 47 | 144.58 | 12.02 |
| 22-Sep-10 | 19941.72 | -0.0030 | 0.00001 | 0.00271 | 48 | 142.04 | 11.92 |
| 23-Sep-10 | 19861.01 | -0.0041 | 0.00002 | 0.00272 | 49 | 139.98 | 11.83 |
| 24-Sep-10 | 20045.18 | 0.0092 | 0.00009 | 0.00281 | 50 | 141.48 | 11.89 |
| 27-Sep-10 | 20117.38 | 0.0036 | 0.00001 | 0.00282 | 51 | 139.34 | 11.80 |
| 28-Sep-10 | 20104.86 | -0.0006 | 0.00000 | 0.00282 | 52 | 136.68 | 11.69 |
| 29-Sep-10 | 19956.34 | -0.0074 | 0.00005 | 0.00288 | 53 | 136.72 | 11.69 |
| 30-Sep-10 | 20069.12 | 0.0056 | 0.00003 | 0.00291 | 54 | 135.67 | 11.65 |
| 1-Oct-10 | 20445.04 | 0.0186 | 0.00034 | 0.00325 | 55 | 148.98 | 12.21 |
| 4-Oct-10 | 20475.73 | 0.0015 | 0.00000 | 0.00325 | 56 | 146.42 | 12.10 |
| 5-Oct-10 | 20407.71 | -0.0033 | 0.00001 | 0.00326 | 57 | 144.34 | 12.01 |
| 6-Oct-10 | 20543.08 | 0.0066 | 0.00004 | 0.00331 | 58 | 143.75 | 11.99 |
| 7-Oct-10 | 20315.32 | -0.0111 | 0.00012 | 0.00343 | 59 | 146.62 | 12.11 |
| 8-Oct-10 | 20250.26 | -0.0032 | 0.00001 | 0.00344 | 60 | 144.61 | 12.03 |
| 11-Oct-10 | 20339.89 | 0.0044 | 0.00002 | 0.00346 | 61 | 143.05 | 11.96 |
| 12-Oct-10 | 20203.34 | -0.0067 | 0.00005 | 0.00351 | 62 | 142.58 | 11.94 |
| 13-Oct-10 | 20687.88 | 0.0237 | 0.00056 | 0.00407 | 63 | 162.79 | 12.76 |
| 14-Oct-10 | 20497.64 | -0.0092 | 0.00009 | 0.00416 | 64 | 163.61 | 12.79 |

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